Factoring

Type I:
Factoring out the GCF (Greatest Common Factor)

Example: $7p^2 + 21p$
$7p(p + 3)$

1. $4w^2 + 2w$

2. $9x^2 + 3x - 18$

3. $15x^2 + 25x + 100$

4. $12x^2 + 6x + 18$

5, $4x^2 + 20x - 12$

6. $-2x^2 + 10x$

7. $x^4 + 2x^3 + x$

8. $12x^2y^3 + 15x^4y^7 - 24x^6y^4$

9. $x^5y^6z^2 + a^3y^9z^5 + w^9y^{10}z^6$

10. $-40x^{10} + 25x^3y^3$

Type II

Factoring $x^2 + bx + c$ where the leading coefficient of x is one.

Rule 1: Check to see if there is a GCF
a) If there is a GCF factor it out! This must be done FIRST.

Rule 2: Examine the trinomial. Find two numbers that multiply to the last number (c) and add to the middle number (b).

Example 1: $x^2 + 10x + 24$
$(x + 6)(x + 4)$

(6)(4) is 24 which = the last number, while 6 + 4 = 10 which is the middle number.

Example 2:	$2x^3 + 6x^2 - 56x$	The terms have a 2x in common. This is the GCF! Factor it out!
	$2x(x^2 + 3x - 28)$	Ask what multiplies to -28, adds to 3?
	$2x(x + 7)(x - 4)$	7 and -4 satisfy this condition

Factor:

1. $x^2 + 6x + 8$

2. $x^2 + 12x + 32$

3. $x^2 + 14x + 40$

4. $x^2 + 8x + 7$

5. $x^2 + 11x + 18$

6. $x^2 - 6x + 8$

7. $x^2 - 7x + 12$

8. $x^2 - 11x + 24$

9. $x^2 - 17x + 72$

10. $x^2 - 14x + 33$

11. $x^2 - 13x + 42$

12. $x^2 - 14x - 32$

13. $x^2 + 3x - 10$

14. $x^2 + 4x - 5$

15. $x^2 + 3x - 28$

16. $x^2 - x - 12$

17. $x^2 - 14x - 15$

18. $2x^2 + 12x - 54$

19. $3x^2 - 3x - 36$

Type 3: Factoring a difference of squares

If two square terms are being subtracted, and they are not like terms, you can factor them as a product of the sum and difference of their square roots.

Rule: $a^2 - b^2 = (a + b)(a - b)$

Example: $x^2 - 1 \rightarrow (x + 1)(x - 1)$

Example: $x^2 - 49 \rightarrow (x + 7)(x - 7)$

1. $x^2 - 4$	2. $x^2 - 9$
3. $x^2 - 100$	4. $x^2 - 81$
5. $x^2 - 36$	6. $x^2 - 121$
7. $2x^2 - 8$ (GCF!) $2(x^2 - 4)$ $2(x + 2)(x - 2)$	8. $3x^2 - 27$ (GCF!)
9. $5x^2 - 125$ (GCF!)	10. $x^4 - 16$
11. $8x^2 - 32$ (GCF!)	12. $9x^2 - 16y^2$
13. $25x^2 - 36y^2$	14. $x^2 - 81y^2$

Type 4: Factoring $ax^2 + bx + c$
The leading coefficient of x is a value not equal to 1

Step 1: Find two numbers that multiply to "a"
Step 2: Find two numbers that multiply to "c"
Step 3: Check the outside product and inside product using FOIL to see if the values add up to the 'b' value. If it does not, switch the order of the "c" values, or try new "c" values, or try new "a" values.

Example: $2x^2 + 9x + 4$
$(2x + 1)(x + 4)$

Break up the $2x^2$ into 2x and x
Ask what multiplies to 4? (1, 4) (4, 1) (2, 2)
Guess and check to see which set of values work
The outside product is 8x
The inside product is 1x
8x + 1x = 9x which is the "b" value middle term.

Example: $3x^2 + 5x + 2$
$(3x + 2)(x + 1)$

Break up $3x^2$ into 3x and x
Break up the "c" value of 2 into 2x1 or 1x2
Check the outside product. Check the inside Product. Does this add up to 5x?
Yes, 3x + 2x = 5x.

Your turn:

1. $4x^2 + 4x - 3$
2. $3a^2 - a - 4$
3. $9a^2 + 18a + 8$
4. $10y^2 + 23y + 12$
5. $15x^2 + 4x - 3$
6. $6y^2 - y - 2$

7. $6x^2 - 5x - 25$

8. $24y^2 - 46y + 10$ (GCF!)

9. $21x^2 + 37x + 12$

10. $18x^2 - 6x - 24$ (GCF!)

11. $9x^2 + 15x + 4$

12. $15y^2 - 19y - 10$

13. $24x^2 - 47x - 2$

14. $20a^2 - 23a + 6$

15. $12x^2 + 34x + 14$ (GCF first!)

Exponents

Rule 1: $a^n \cdot a^m = a^{n+m}$

Same base variable being multiplied, add the exponents

Example: $x^6 \cdot x^3 = x^{6+3} = x^9$

1. $x^5 \cdot x^4$

2. $y^5 \cdot y^7 \cdot y^3$

3. $(x^2y)(x^3y^4)$

4. $(2^2)(2^3) = (\quad) =$ ______

Rule 2: $(a^n)^m = a^{n \cdot m}$

One base with a power raised to another power, multiply the powers!

Example: $(a^4)^5 = a^{20}$

Example: $(3xy^4z^5)^2 = 9x^2y^8z^{10}$

1. $(a^3b^4)^5 =$

2. $(3^2)^2 =$

3. $(2ab^2c^6)^3 =$

4. $(-4a^6bc^4)^3 =$

Rule 3: $\frac{x^a}{x^b} = x^{a-b}$

When you have the same variable being divided, you subtract the exponents

Example: $\frac{x^5}{x^2} = x^{5-3} = x^2$

1. $\frac{x^8}{x^2}$

2. $\frac{x^{12}}{x^{11}}$

3. $\frac{x^4 y^7}{x^3 y}$

4. $\frac{6a^3 b^7 c^{11}}{12a\ b^2 c^3}$

5. $\frac{16x^{12} y^5 z^2}{14x^3 y^5}$

6. $\frac{-36xy^{12} z^9}{15y^{-3} z^{-1}}$

Rule 4: $\left(\frac{a}{b}\right)^n$ = $\frac{a^n}{b^n}$

When a fraction is being raised to a power, you distribute that power to the numerator and denominator of the fraction.

Example: $\left(\frac{5}{6}\right)^2$ = $\frac{5^2}{6^2} = \frac{25}{36}$

1. $\left(\frac{3}{4}\right)^2$ =

2. $\left(\frac{2a^2}{3b^3}\right)^2$ =

3. $\left(\frac{4ab^4}{2a^4 b^2}\right)^3$ =

Note: Make sure you distributed the exponent to the whole numbers!

Rule 5: $a^0 = 1$
Anything raised to the zero power = 1

1. $10^0 =$ 2. $32^0 =$ 3. $(x^2)^0 =$ 4. $2(3x^3)^0 =$

Rule 6: Negative Exponents

$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

If a term has a negative exponent in the numerator, you move it to the denominator and make the exponent positive.
If a term has a negative exponent in the denominator, you move it to the numerator and make the exponent positive.

Example: $3^{-1} = \frac{1}{3^1}$

Example: $\frac{3a^{-5}b^4}{a^2b^{-3}} = \frac{3b^4b^3}{a^2a^5} = \frac{3b^7}{a^7}$

1. x^{-2}

2. $(4y)^{-3}$

3. $3a^{-2}$

4. $\left(\frac{9abc}{x}\right)^{-1}$

5. $\left(\frac{3x^2y}{4^{-4}y^{-2}}\right)^{-2}$

6. $\frac{3^{-1}x^4y^3z^{-2}}{2^2x^{-5}y^{-6}z^{-8}}$

Simplify with only positive exponents in your final answer:

1. $\dfrac{4x^{-3}y^{4}z}{12x^{5}y^{6}z^{-7}}$

2. $\left(\dfrac{3x^{-4}y^{-10}}{27x^{-1}y}\right)^{-1}$

3. $\left(\dfrac{2x^{10}y^{4}z^{2}}{6xyz^{5}}\right)^{2}$

4. $\dfrac{\left(-4a^{-3}b^{2}\right)^{3}}{\left(2ab^{6}\right)^{2}}$

5. $\dfrac{8^{-2}m^{-6}n^{7}}{-6m^{-9}n^{4}}$

6. $\left(\dfrac{5^{-2}a^{6}b^{12}c}{10a^{-1}b^{-8}c^{6}}\right)^{-2}$

Radicals

Rule: Find the largest perfect square that goes into the number in the radical

1. $\sqrt{8}$ = $\sqrt{4}\sqrt{2} = 2\sqrt{2}$

2. $\sqrt{28}$

3. $\sqrt{48}$

4. $\sqrt{24}$

5. $\sqrt{72}$

6. $\sqrt{12}$

7. $\sqrt{98}$

8. $\sqrt{63}$

9. $\sqrt{300}$

10. $\sqrt{125}$

11. $\sqrt{45}$

12. $\sqrt{40}$

Multiplying Radicals

Multiply and Simplify Your Final Answer

1) $\sqrt{5} * \sqrt{2} =$ ____________

2) $\sqrt{6} * \sqrt{2} =$ ____________

3) $\sqrt{3} * \sqrt{3} =$ ____________

4) $\sqrt{16} * \sqrt{2} =$ ____________

5) $2\sqrt{2} * 3\sqrt{3} =$ ____________

6) $6\sqrt{5} * 2\sqrt{5} =$ ____________

7) $\sqrt{7} * 2\sqrt{3} =$ ____________

8) $\sqrt{6} * 3\sqrt{3} =$ ____________

9) $4\sqrt{6} * 2\sqrt{4} =$ ____________

10) $12\sqrt{3} * 2\sqrt{3} =$ ____________

11) $7\sqrt{2} * 2\sqrt{7} =$ ____________

12) $4\sqrt{11} * 6\sqrt{3} =$ ____________

Dividing with Radicals

Radicals **<u>CANNOT</u>** be left in the denominator (bottom) of a fraction!

Sometimes we will see situations where there is a radical in the denominator and we will have to get rid of it.

To get rid of it, multiply the numerator (top) and denominator (bottom) by the radical that we are trying to get rid of

1) $\frac{2}{\sqrt{3}}$

2) $\frac{1}{\sqrt{5}}$

3) $\frac{4}{\sqrt{2}}$

4) $\frac{\sqrt{5}}{\sqrt{2}}$

5) $\frac{\sqrt{3}}{\sqrt{6}}$

6) $\frac{2\sqrt{7}}{\sqrt{5}}$

7) $\frac{3\sqrt{11}}{2\sqrt{3}}$

8) $\frac{9\sqrt{2}}{\sqrt{2}}$

Multiply and simplify where necessary:

1. $\sqrt{6} \cdot \sqrt{2}$

2. $\sqrt{10} \cdot \sqrt{5}$

3. $\sqrt{12} \cdot \sqrt{3}$

4. $\sqrt{5} \cdot \sqrt{7}$

5. $\sqrt{3} \cdot \sqrt{6}$

6. $2\sqrt{3} \cdot 5\sqrt{7}$

7. $4\sqrt{6} \cdot 7\sqrt{5}$

8. $2\sqrt{5} \cdot 5\sqrt{4}$

9. $10\sqrt{3} \cdot 7\sqrt{2}$

10. $2\sqrt{10} \cdot \sqrt{6}$

11. $3\sqrt{2} \cdot \sqrt{22}$

12. $5\sqrt{6} \cdot 3\sqrt{2}$

13. $10\sqrt{12} \cdot 3\sqrt{2}$

14. $2\sqrt{8} \cdot 3\sqrt{8}$

Divide and simplify:

15. $\dfrac{1}{\sqrt{7}}$

16. $\dfrac{3}{\sqrt{5}}$

17. $\dfrac{12}{\sqrt{6}}$

18. $\dfrac{10}{\sqrt{3}}$

19. $\dfrac{2\sqrt{3}}{\sqrt{3}}$

20. $\dfrac{5}{\sqrt{15}}$

21. $\dfrac{\sqrt{3}}{\sqrt{15}}$

22. $\dfrac{4\sqrt{5}}{\sqrt{8}}$

23. $\dfrac{6\sqrt{10}}{\sqrt{2}}$

24. $\dfrac{2\sqrt{3}}{4\sqrt{2}}$

25. $\dfrac{5\sqrt{6}}{3\sqrt{5}}$

26. $\dfrac{3\sqrt{6}}{8\sqrt{2}}$

Slope

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Find the slope of the line passing through the following points:

1. (4, 3) (7, 1)

   m = $\frac{3-1}{4-7} = \frac{2}{-3}$

2. (0, 5) (3, 9)

3. (-3, 4) (2, 11)

4. (-5, -8) (-1, -4)

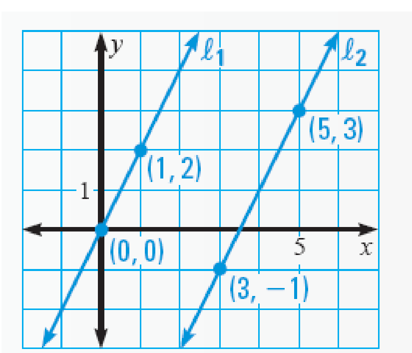
5. (3, 0) (2, 0)

6. (4, 0) (0, 6)

7. What is the slope of a horizontal line?

8. What is the slope of a vertical line?

9. Find the slope of l_1 and l_2

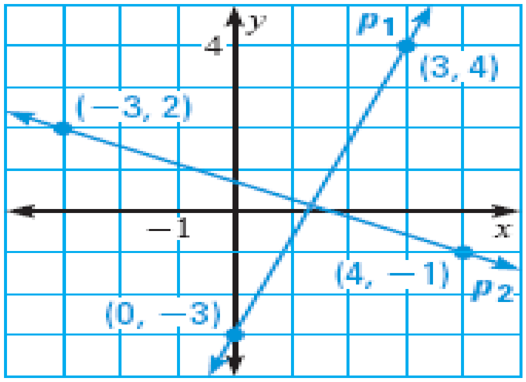


L_1 = __________

L_2 = __________

Are these lines parallel? ________

10. Are the lines parallel, perpendicular, or neither?



Slope of P_1 = ________

Slope of P_2 = ________

11. **MULTIPLE CHOICE** Which lines are parallel?

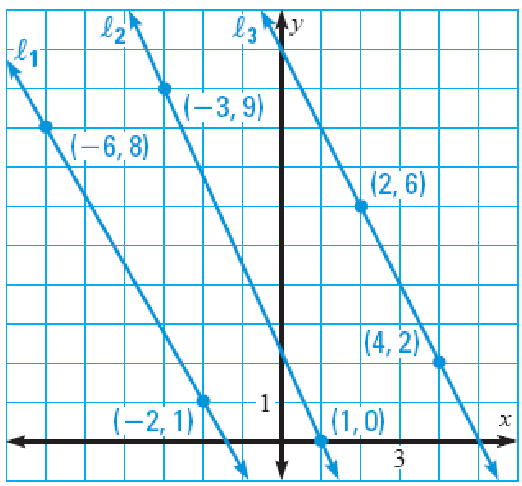
Ⓐ $\ell_1 \parallel \ell_2$

Ⓑ $\ell_2 \parallel \ell_3$

Ⓒ $\ell_1 \parallel \ell_3$

Ⓓ None

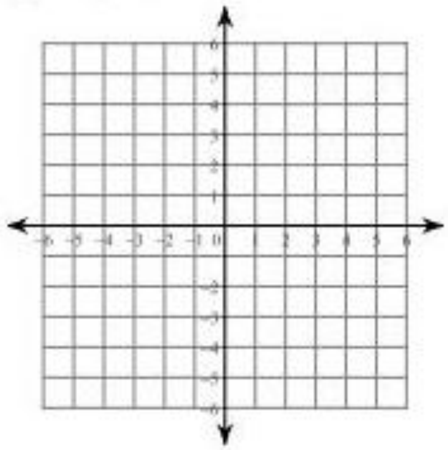
Ⓔ All 3



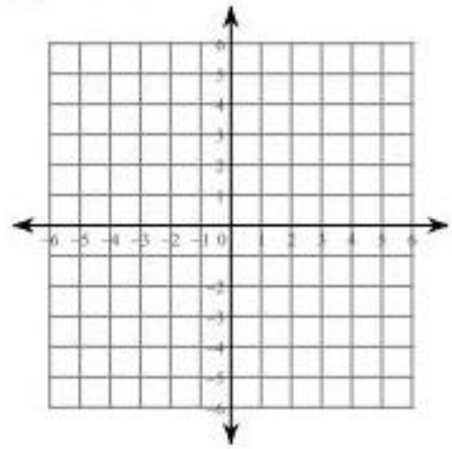
Graphing lines

Sketch the graph of each line.

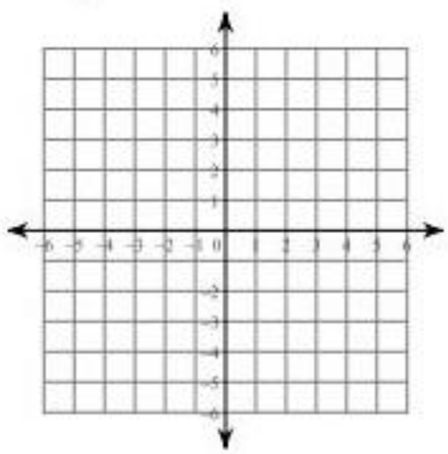
1) $y = -3x + 3$



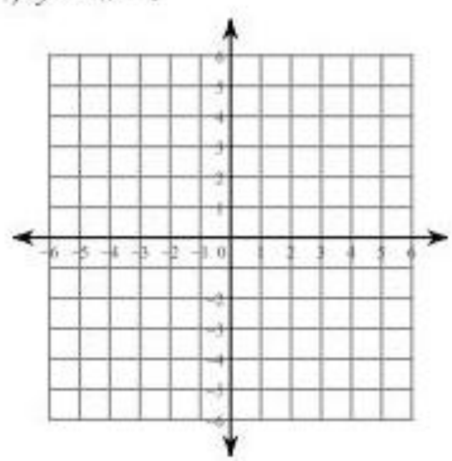
2) $y = x + 5$



3) $y = \frac{5}{3}x - 4$



4) $y = -x - 1$



11) through: $(4, -1)$, perp. to $y = \frac{1}{3}x - 3$

12) through: $(3, 1)$, perp. to $y = 8x - 4$

Write the slope-intercept form of the equation of the line through the given point with the giver

13) through: $(1, 4)$, slope $= -\frac{1}{4}$

14) through: $(3, -5)$, slope $= -\frac{8}{5}$

15) through: $(3, 1)$, slope $= -4$

16) through: $(2, 5)$, slope $= -5$

17) through: $(2, 2)$, slope $= 6$

18) through: $(3, -3)$, slope $= \frac{2}{5}$

Write the slope-intercept form of the equation of the line through the given points.

19) through: $(0, -3)$ and $(-5, -3)$

20) through: $(-1, -3)$ and $(0, -4)$

21) through: $(0, -1)$ and $(1, -4)$

22) through: $(0, -1)$ and $(-3, 3)$

23) through: $(5, -2)$ and $(0, 0)$

24) through: $(3, 0)$ and $(0, -2)$